Do you know why $\sqrt{3}$ isn't rational?

We will now show that $\sqrt{3}$ is irrational by supposing that we could write $\sqrt{3}$ as a fraction of counting numbers and by deducing that this could not be the case. So suppose that

$$\sqrt{3} = \frac{A}{B}$$

where A and B are counting numbers. Then

$$3 = \left(\frac{A}{B}\right)^2$$

so $$3 = \frac{A^2}{B^2}$$

By multiplying both sides of this equation by B^2, we have

$$3 \cdot B^2 = A^2$$

Now imagine factoring A and B into product of prime numbers.

Then A^2 has an even number of prime factors because it has twice as many prime factors as A does. For example, if A were 42, then

$$A = 2 \cdot 3 \cdot 7$$

and $$A^2 = 2 \cdot 3 \cdot 7 \cdot 2 \cdot 3 \cdot 7$$

which has 6 prime factors, twice as many as A. Similary $3 \cdot B^2$ has an odd number of prime factors because it has one more than twice as many prime factors as B. Fo example, if B were 35, then

$$B = 5 \cdot 7$$

and $$3 \cdot B^2 = 3 \cdot 5 \cdot 7 \cdot 5 \cdot 7$$

which has 5 prime factors, one more than twice as many as B.

But if $3 \cdot B^2$ has an odd number of prime factors and A^2 has an even number of prime factors then it cannot be the case that

$$3 \cdot B^2 = A^2$$

Therefore, it cannot be the case that

$$\sqrt{3} = \frac{A}{B}$$

where A and B are counting numbers. Thus, $\sqrt{3}$ is irrational.